

# Centrality Dependence of Direct Photons in Au+Au Collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV

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We calculate the spectra of high energy photons emitted in relativistic Au+Au collisions for various centralities and compare to data recently collected at the Relativistic Heavy Ion Collider by the PHENIX collaboration. Our results for photons from primary hard scatterings and photons from interactions of jets with the medium are consistent with the measurements of neutral pion and direct photon production in  $p + p$  collisions and give a good description of direct photon spectra measured in Au+Au collisions. The contribution of photons from jet-to-photon conversion in the medium can be as large as the photon yield from hard scatterings in the momentum range  $p_T \approx 2 \dots 6$  GeV/ $c$ . We show that this novel mechanism is not ruled out by any existing data.

High energy nuclear collisions are studied to understand the properties of strongly interacting matter at the highest energy densities and to search for the transition of hadronic matter to a quark-gluon plasma [1]. Electromagnetic probes are excellent tools to gather information about the dense and hot medium created in such collisions. The mean free path of photons is much larger than the typical size of the emerging fireball, allowing them to escape nearly unperturbed once created in the system [2]. Since photons are emitted throughout the history of the fireball, one expects to obtain information that is complementary to that gained from the measurement of hadrons, which are subject to strong final-state interactions and are reflecting the latest stage of the collision.

Thermal radiation of photons can occur both from a quark-gluon plasma [3, 4] and from a hot hadronic gas [5, 6]. Because the photon spectrum reflects the thermal distribution of the emitting matter, it can serve as a thermometer. On the other hand, high energy photons are most likely to be created either in hard processes between partons in the colliding nuclei [7], or in final-state interactions of high energy jets [8]. Photons from jets naturally carry information about the medium they traverse and hence can serve as a medium probe.

The latter process was introduced in our earlier work [8] where we predicted the existence of a medium induced component in the photon spectrum in the intermediate  $p_T$  range. In this novel mechanism, a hard quark passing through the medium converts into a photon carrying most of its transverse momentum. Recently Zakharov has developed the related concept of modified photon bremsstrahlung of jets in the medium [9]. An up-to-date calculation of photon emission including such jet-medium interactions can be found in the work of Turbide *et al.* [10]. The general arguments given for photons in this work also apply to the emission of lepton pairs initiated by quark jets in the plasma [11].

Relativistic nuclear collisions at different centralities result in the formation of systems of different sizes, initial temperatures and life times. Assuming that the yield of

processes involving large momentum transfer (hard processes) scale as the number of collisions  $N_{\text{coll}}$ , while those with small momentum transfer (soft processes) scale as the number of participants  $N_{\text{part}}$ , one can predict the relative importance of the two contributions as a function of centrality (see e.g. [12]). The recently published data on single photon production in  $p + p$  collisions [13] at  $\sqrt{s_{\text{NN}}} = 200$  GeV [13] and the centrality dependence of the photon yield in Au+Au collisions measured by PHENIX at the Relativistic Heavy Ion Collider (RHIC) [14] is our first chance to obtain an understanding of the different photon sources.

Here, we shall address the question whether the data allow us to draw conclusions about the relative contributions from different photon production mechanisms to the direct photon spectrum in Au+Au collisions. The data is still limited by (statistical and systematic) uncertainties, but the results represent a remarkable achievement because of the experimental challenge to subtract the background of photons from secondary hadronic decays, especially neutral pions. Fortunately, in Au+Au collisions at RHIC this background is reduced at intermediate and high transverse momenta  $p_T > 2$  GeV/ $c$  by the large final state suppression (commonly called “jet quenching”) for pions and other hadrons [15, 16]. Thus, while the low- $p_T$  part of the photon spectrum remains clouded by the large hadronic background, this is an invitation to look for electromagnetic signals of a quark gluon plasma at intermediate and high  $p_T$  [8, 17].

Let us briefly recall the photon sources that are important at  $p_T > 2$  GeV/ $c$ . We define prompt photons as those coming from hard Compton and annihilation processes (e.g.  $q + g \rightarrow q + \gamma$  and  $q + \bar{q} \rightarrow g + \gamma$ ) and those emitted from jets after the hard interaction as bremsstrahlung (e.g.  $q \rightarrow q + \gamma$ ). A baseline for these processes can be obtained by studying  $p + p$  and  $p + \bar{p}$  interactions. Besides an obvious scaling from  $p + p$  collisions to Au+Au collisions with the number  $N_{\text{coll}}$ , and an isospin correction due to the presence of  $p + n$  and  $n + n$  collisions, Compton and annihilation spectra are expected to be altered

only marginally by initial state nuclear effects. Neutral pion spectra measured in  $d$ +Au collisions indicate that such effects from shadowing and  $k_T$ -broadening are small [18]. The hard process itself is pointlike on typical scales of the medium and therefore remains unaffected. Direct photon spectra measured in  $d$ +Au could help to quantify the role of initial state effects.

Bremsstrahlung emission from jets, on the other hand, takes place on more extended distances and thus can be affected by final state interactions. Energy loss effects would then be noticeable as a suppression of bremsstrahlung [19], while secondary scattering in the medium could initiate additional bremsstrahlung or modify bremsstrahlung by coherence effects [9]. In our work we assume that the total photon yield from jets can be described by the superposition of the usual vacuum bremsstrahlung with an additional contribution from jets annihilating and Compton scattering in the medium as described in [8].

Photons from secondary hard scatterings in the pre-equilibrium phase have been studied elsewhere and could contribute to photons at intermediate and high  $p_T$  [20]. In order to minimize possible effects from neglecting pre-equilibrium emission, we assume a very early thermalization time,  $\tau_0 \approx 0.15$  fm/c, in central collisions, not much larger than the formation time of minijets. An early thermalization of the matter created in Au+Au collisions at RHIC is supported by several observations, especially the large elliptic flow. We also assume that the matter is chemically equilibrated at the initial time. Secondary collisions are then either between thermalized partons or between hard scattered partons and the medium, which are both taken into account here. Similar strategies for dealing with the uncertainties of pre-equilibrium emission have been invoked elsewhere [21].

We now recall the specifics of the jet-medium interaction process. We denote the momenta of the jet, the thermal parton, and the photon by  $\mathbf{p}_{\text{jet}}$ ,  $\mathbf{p}_{\text{th}}$  and  $\mathbf{p}_\gamma$ , respectively. The leading order QCD Compton and annihilation cross sections are peaked in the forward and backward directions. In the laboratory frame we have  $|\mathbf{p}_{\text{jet}}| \gg |\mathbf{p}_{\text{th}}| \sim T$ , where  $T$  is the temperature of the plasma. For high energy photons, i.e.  $|\mathbf{p}_\gamma| \gg T$ , this implies that  $|\mathbf{p}_\gamma| \approx |\mathbf{p}_{\text{jet}}|$ . This justifies calling the process a conversion of a jet (or more precisely: a quark initiating a jet) into a photon with similar momentum [8]. Only quark jets contribute to this process, because the gluon-photon scattering cross section is not peaked at forward angles.

The rate of photon production by annihilation and Compton scattering of jets in the medium can be approximated as [8]

$$\frac{E_\gamma dN}{d^3\mathbf{p}_\gamma d^4x} = \frac{\alpha\alpha_s}{4\pi^2} \sum_q e_q^2 f_q(\mathbf{p}_\gamma, x) T^2 \left[ \ln \frac{4E_\gamma T}{m_{\text{th}}^2} + C \right] \quad (1)$$

where  $C = -1.916$ ,  $m_{\text{th}}^2 = g^2 T^2/6$  and  $\alpha_s = g^2/(4\pi)$  is the strong and  $\alpha$  the electromagnetic coupling.  $q$  denotes all light quark and antiquark species with charge  $e_q$ , and  $f_q$  is the distribution of minijets of flavor  $q$ . It is worth emphasizing that the conversion property of the process is reflected in (1) by the fact that the photon spectrum is directly proportional to the jet spectrum  $f_q$ . Calculations without some of the simplifying approximations [8] that lead to the expression in Eq. (1) produce only small modifications of the photon yield [10].

In order to calculate the yield from thermal emission and jet conversion, we need to model the space-time evolution of the fireball. We want to compare with data taken at midrapidity, hence we assume the boost invariant Bjorken scenario for the longitudinal expansion. In addition we neglect the marginal transverse expansion of the plasma on the jet-photon conversion part, i.e. during the short time jets are propagating through the QGP. But the transverse expansion is fully taken into account for the thermal contribution.

We integrate over the conversion vertex  $x$  in (1) in terms of the space-time rapidity  $\eta$ , the radial coordinate  $\mathbf{r}$  (with  $r = |\mathbf{r}|$ ) at which the jet is emitted, and the proper time  $\tau$ , so that  $d^4x = \tau d\tau d\eta d^2r$ . We parameterize momenta  $\mathbf{p}$  by their rapidity  $y$  and transverse momentum  $\mathbf{p}_T$ , ( $p_T = |\mathbf{p}_T|$ ). Starting from the yield  $dN_{\text{jet}}/(d^2p_T dy)$  of quarks initiating a jet, the distribution of hard partons in a boost-invariant scenario is given by [8, 22]

$$f_q(\mathbf{p}, x) = \frac{(2\pi)^3}{g_q \pi R_T^2 \tau p_T} \frac{dN_{\text{jet}}}{d^2p_T dy} \rho(\mathbf{r}) \delta(y - \eta) \quad (2)$$

where  $R_T$  is the radius of the fireball,  $g_q = 6$  the spin-color degeneracy factor of parton  $q$ , and  $\rho$  is a transverse profile function normalized to one. For simplicity, we will take the profile to be azimuthally symmetric with area density  $\rho(r) = 2(1 - r^2/R_T^2)\Theta(R_T - r)$  and transverse radius  $R_T = 1.2(N_{\text{part}}/2)^{1/3}$ .

The number of participants,  $N_{\text{part}}$ , is a function of centrality, but we neglect the azimuthal asymmetry in non-central collisions. This approximation does not allow us to predict the azimuthal distribution of photons, but it introduces only a small error for the yield integrated over the azimuthal angle. The jet spectrum for an arbitrary impact parameter  $b$  is obtained by scaling

$$\frac{dN_{\text{jet}}}{d^2p_T dy}(b) = \frac{T_{\text{AA}}(b)}{T_{\text{AA}}(0)} \times \frac{dN_{\text{jet}}}{d^2p_T dy}(b=0). \quad (3)$$

This expression shows that the photon yield due to jet-photon conversion scales as a product of the nuclear thickness and a term dependent on the initial conditions, like temperature and size, and on the history of the evolution of the fireball. A measurement of the photon spectra at different centralities could thus help reveal this information, as the primary hard photons will scale with the nuclear thickness  $T_{\text{AA}}$  alone.

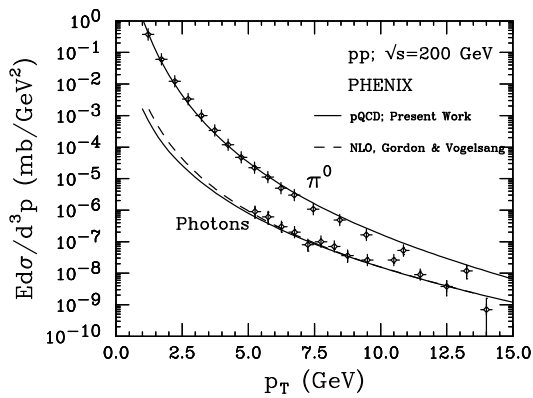


FIG. 1: Invariant cross section  $E d\sigma/d^3p$  for  $\pi^0$  and  $\gamma$  production in  $p + p$  collisions at  $\sqrt{s} = 200$  GeV as a function of transverse momentum  $p_T$  at  $y = 0$ . Data points are from the PHENIX experiment [13, 28]. We also show a full NLO calculation [27].

We use a leading order perturbative calculation of jets and of direct Compton and annihilation photons [7], using CTEQ5L parton distributions [23]. For gold nuclei EKS98 corrections of the nuclear parton distributions [24] are taken into account. Higher order perturbative effects are accounted for by a phenomenological  $K$ -factor, which is known to depend weakly on the transverse momentum  $p_T$  at RHIC energies [25] and is here assumed to be constant.

Vacuum photon bremsstrahlung is obtained by convoluting the jet distributions with the photon fragmentation functions given in [7] ( $\Lambda = 200$  MeV), and we use KKP fragmentation functions [26] for the yield of neutral pions. We calculate the  $\pi^0$  and photon cross sections at midrapidity for  $p + p$  and compare to the values measured by PHENIX. This allows us to fix the  $K$  factors for jet production,  $K_{\text{jet}}$ , and direct Compton and annihilation photons,  $K_\gamma$ , independently. We obtain a good description of the  $p + p$  data with  $K_{\text{jet}} = 1.8$  and  $K_\gamma = 1.5$ , see Fig. 1. These values agree with those reported in ref. [10] on the basis of a next-to-leading order (NLO) perturbative QCD calculation and our results also match the NLO calculation by Gordon and Vogelsang [27].

Once the  $K$ -factors are fixed we apply them to calculate the yield of direct photons in Au+Au collisions. The prompt photon yield in Au+Au is close to that in  $p + p$  scaled with  $T_{AA}$ , due to small shadowing corrections. Note, however, that there is an effect from the admixture of  $p + n$  and  $n + n$  collisions with different photon yields. Therefore a simple scaling of  $p + p$  results, as often invoked by the experimental collaborations for comparison, introduces a small but systematic bias of the photon yield from hard processes towards larger values.

Next we use the jet yields in Au+Au to calculate the photons from jet-photon conversions using (1) and (2).

Centrality	0-10%	10-20%	20-30%	30-40%	40-50%
$N_{\text{part}}$	326.2	234.5	166.0	114.0	75.0
$T_{AA}$ [1/mb]	22.75	14.35	8.00	5.23	2.86
$\tau_0$ [fm/c]	0.151	0.166	0.182	0.201	0.221
$T_0$ [MeV]	0.434	0.395	0.361	0.328	0.297

TABLE I: Number of participants  $N_{\text{part}}$ , nuclear thickness factor  $T_{AA}$ , initial time  $\tau_0$  and initial temperature  $T_0$  for different centrality bins.

For the fireball we assume a thermally and chemically equilibrated plasma at some early time  $\tau_0$  with initial temperature  $T_0$ . The product  $T^3\tau$  is conserved during the isentropic longitudinal expansion, and its value can be fixed by the total hadron multiplicity  $dN_h/dy$ . The initial conditions are determined by imposing the thermalization condition  $\tau_0 \approx 1/(3T_0)$ . Table I lists the initial times and temperatures inferred from the charged hadron multiplicity  $dN/d\eta$  measured by PHENIX [15] for different centrality bins. Nuclear thickness factors and number of participants are also given.

The transverse profile of the initial temperature is fixed by assuming that the energy density scales with the square of the nuclear thickness factors, so that  $T_i(r) = T_0 [2(1 - r^2/R_T^2)]^{1/4}$ . This has been found to work well for the study of the centrality dependence of hadronic spectra [29] as well as lepton pair production [30], and should be sufficient for our purpose. Starting from the initial time  $\tau_0$ , we integrate (1) to either the time  $\tau_f$ , when the medium has cooled to the critical temperature  $T_f = 160$  MeV, or to the time when the jet reaches the boundary of the fireball, whichever comes first.

For the calculation of thermal photons we assume that the thermally and chemically equilibrated quark-gluon plasma, which undergoes a boost-invariant longitudinal and an azimuthally symmetric transverse expansion [31], converts to a hadronic gas below  $T_c$ . The production of photons from the deconfined phase is calculated using the complete leading order emission rate [4]. The latest results of Turbide *et al.* [6] for photon emission from hadronic matter are used.

Figs. 2, 3, 4 and 5 present our results for photon yields at midrapidity in Au+Au collisions as a function of photon transverse momentum in four different centrality bins: 0-10%, 10-20%, 20-30% and 40-50%. Both prompt photons and photons from jet-plasma interactions are shown, and the sum is compared to PHENIX data [14]. For completeness we also show our result for thermal photons. The agreement with data is generally quite good for all centrality bins. This is also true for the other centrality bins, up to the 60-70% bin, which are not shown here.

As already pointed out in [8], the jet-photon conversion spectrum is falling off with  $p_T$  faster than the spectrum of prompt photons, a consequence of the additional

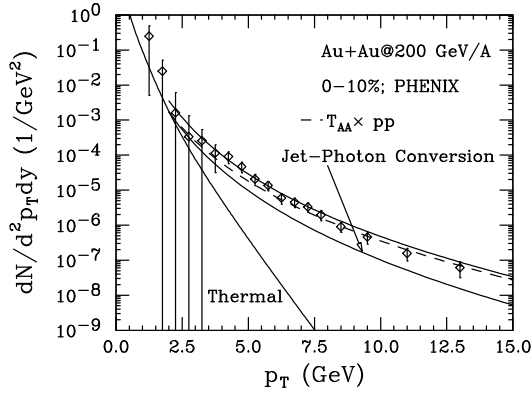


FIG. 2: Photon yield  $dN/(d^2 p_T dy)$  as a function of  $p_T$  for  $y = 0$  in central (0-10%) Au+Au collisions at  $\sqrt{s} = 200$  GeV. We show primary hard photons (dashed), jet-photon conversion (solid and labelled) and the sum of both (upper-most solid curve). Thermal photons are also shown (solid and labelled). Data are from the PHENIX collaboration [14].

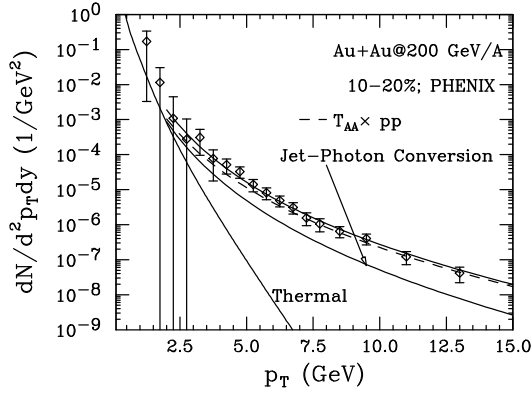


FIG. 3: The same as Fig. 2 but for the 10-20% centrality bin.

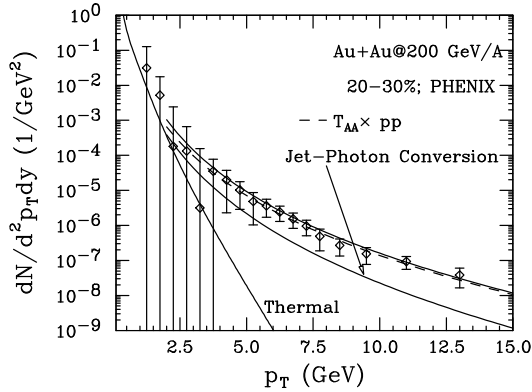


FIG. 4: The same as Fig. 2 but for the 20-30% centrality bin.

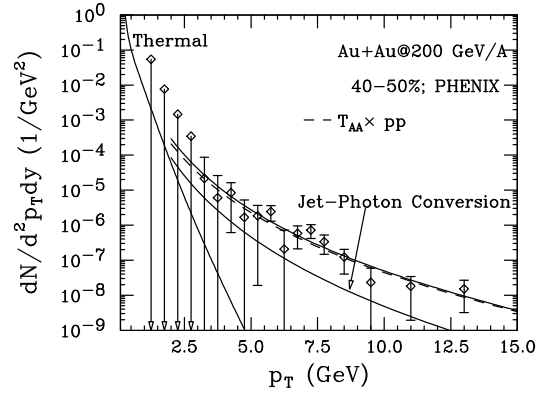


FIG. 5: The same as Fig. 2 but for the 40-50% centrality bin.

hard momentum transfer with a propagator  $\sim 1/p_T^2$ . Interestingly the jet-photon conversion yield is as large as both the primary hard photon yield and the thermal yield around  $p_T = 2$  GeV/c in central Au+Au collisions. Unfortunately, the quality of data in this region is still poor. Between 4 and 6 GeV/c, where the data provide a better constraint, jet-medium photons still add roughly a 50% contribution on top of primary hard photons.

It has been argued that photons from primary hard processes alone are sufficient to describe the Au+Au data [14, 21] and that no additional contribution is needed. The first statement is correct, to a certain extent, as can be seen in Fig. 2. However, its value is limited by the size of the experimental error bars. We have demonstrated here that a calculation taking into account jet-photon conversion in a consistent manner also leads to a result which is compatible with present data. Caution should be exercised when isospin effects are omitted in the discussion.

Energy loss effects for jets before they convert into photons have been investigated by Turbide *et al.* [10]. The net effect on the yield of photons from jet-medium interactions is found to be small, about 20%. This is because most photons are emitted at a time when the plasma is still hot. Jets have then traveled only a short distance through the plasma and have not lost a significant amount of energy.

The centrality dependence of the data is well described by our calculations. While direct photons scale with the number of collisions  $N_{\text{coll}}$ , photons from jet-medium interactions exhibit a slightly stronger scaling. The relative importance of this process thus decreases in more peripheral collisions. This can also be seen in Fig. 6 where we plot the nuclear modification factor

$$R_{AA} = \frac{dN_{AA}/dy|_{p_T > 6 \text{ GeV}/c}}{N_{\text{coll}} dN_{pp}/dy|_{p_T > 6 \text{ GeV}/c}} \quad (4)$$

calculated from the photon spectra integrated for  $p_T > 6$  GeV/c vs centrality. We show our calculation and a ver-

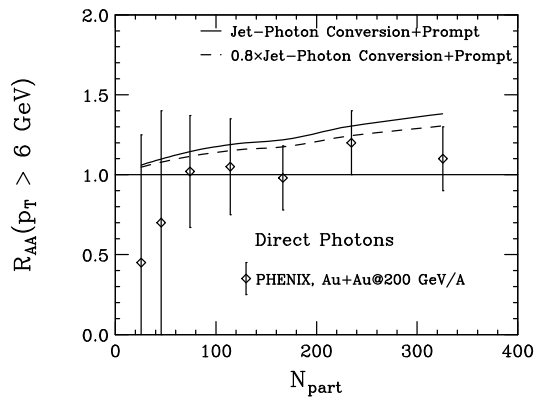


FIG. 6: The nuclear modification factor  $R_{AA}$  for direct photons and  $p_T > 6$  GeV/c as defined in (4) as a function of the number of participants  $N_{part}$  (solid line). We also show a calculation effectively taking into account energy loss of jets before convert into photons (dashed line). Data are taken from PHENIX collaboration [14].

sion taking into account an effective 20% energy loss of jets before conversion into photons and compare with PHENIX data [14]. Due to the large error bars the data is compatible with 1 for central and midperipheral collisions, in accordance with a binary collision scaling from  $p + p$  collisions. Our calculation is compatible with the data as well, and predicts a slight rise of  $R_{AA}$  with centrality. Our results show that the importance of the contribution from jet-photon conversion to the total photon spectrum grows below  $p_T = 6$  GeV/c. A high statistics measurement of  $R_{AA}$  in this region would be very useful.

In summary, we have calculated the photon spectrum in Au+Au collisions at RHIC energies resulting from primary hard photons, jets interacting with the medium and thermal radiation. The calculations are consistent with neutral pion and photon spectra measured in  $p + p$ . We obtain a good description of the  $p_T$  dependence and centrality dependence of photon production in Au+Au. The contribution from jet-photon conversions can be as large as 100% (50%) that of primary hard photons at 2 (5) GeV/c in accordance with data measured by PHENIX.

At the Large Hadron Collider (LHC), the importance of the jet-photon conversion process will be significantly enhanced. It has been shown that it will be the dominant contribution below  $p_T \approx 12$  GeV/c [8, 10].

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[1] J. W. Harris and B. Müller, *Ann. Rev. Nucl. Part. Sci.* **46**, 71 (1996).  
 [2] E. L. Feinberg, *Nuovo Cim. A* **34**, 391 (1976).  
 [3] J. I. Kapusta, P. Lichard and D. Seibert, *Phys. Rev.*

*D* **44**, 2774 (1991); *Erratum ibid.* *D* **47**, 4171 (1993);  
 R. Baier, H. Nakkagawa, A. Niegawa and K. Redlich, *Z. Phys. C* **53**, 433 (1992); P. K. Roy, D. Pal, S. Sarkar, D. K. Srivastava and B. Sinha, *Phys. Rev. C* **53**, 2364 (1996); P. Aurenche, F. Gelis, R. Kobes and H. Zaraket, *Phys. Rev. D* **58**, 085003 (1998).  
 [4] P. Arnold, G. D. Moore and L. G. Yaffe, *JHEP* **0112**, 009 (2001).  
 [5] L. Xiong, E. V. Shuryak and G. E. Brown, *Phys. Rev. D* **46**, 3798 (1992); V. V. Goloviznin and K. Redlich, *Phys. Lett. B* **319**, 520 (1993); C. Song, *Phys. Rev. C* **47**, 2861 (1993).  
 [6] S. Turbide, R. Rapp, and C. Gale, *Phys. Rev. C* **69**, 014903 (2004).  
 [7] J. F. Owens, *Rev. Mod. Phys.* **59**, 465 (1987).  
 [8] R. J. Fries, B. Müller and D. K. Srivastava, *Phys. Rev. Lett.* **90**, 132301 (2003).  
 [9] B. G. Zakharov, *JETP Lett.* **80**, 1 (2004).  
 [10] S. Turbide, C. Gale, S. Jeon and G. D. Moore, *preprint hep-ph/0502248*.  
 [11] D. K. Srivastava, C. Gale and R. J. Fries, *Phys. Rev. C* **67**, 034903 (2003).  
 [12] See e.g., S. S. Adler *et al.* [PHENIX Collaboration] *Phys. Rev. C* **71**, 034908 (2005); *Erratum-ibid.* *C* **71**, 049901 (2005); and references there-in.  
 [13] S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. D* **71**, 071102 (2005).  
 [14] S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **94**, 23201 (2005).  
 [15] S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **91**, 072301 (2003); *Phys. Rev. C* **69**, 034910 (2004).  
 [16] J. Adams *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **91**, 172302 (2003).  
 [17] C. Gale, T. C. Awes, R. J. Fries and D. K. Srivastava, *J. Phys. G* **30**, S1013 (2004).  
 [18] S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **91**, 072303 (2003).  
 [19] S. Jeon, J. Jalilian-Marian and I. Sarcevic, *Phys. Lett. B* **562**, 45 (2003).  
 [20] R. Baier, M. Dirks, K. Redlich and D. Schiff, *Phys. Rev. D* **56**, 2548 (1997); D. K. Srivastava and K. Geiger, *Phys. Rev. C* **58**, 1734 (1998); S. A. Bass, B. Müller and D. K. Srivastava, *Phys. Rev. Lett.* **93**, 162301 (2004).  
 [21] D. d'Enterria and D. Peressounko, *preprint nucl-th/0503054*.  
 [22] Z. W. Lin and M. Gyulassy, *Phys. Rev. C* **51**, 2177 (1995); *Erratum-ibid.* *C* **52**, 440 (1995).  
 [23] H. L. Lai *et al.* [CTEQ Collaboration], *Eur. Phys. J. C* **12**, 375 (2000).  
 [24] K. J. Eskola, V. J. Kolhinen and C. A. Salgado, *Eur. Phys. J. C* **9**, 61 (1999).  
 [25] G. G. Barnafoldi, G. I. Fai, P. Levai, G. Papp and Y. Zhang, *J. Phys. G* **27**, 1767 (2001).  
 [26] B. A. Kniehl, G. Kramer and B. Pötter, *Nucl. Phys. B* **582**, 514 (2000).  
 [27] L. E. Gordon and W. Vogelsang, *Phys. Rev. D* **48**, 3136 (1993); *Phys. Rev. D* **49**, 170 (1994).  
 [28] S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **91**, 241803 (2003).  
 [29] D. K. Srivastava, *Phys. Rev. C* **64**, 064901 (2001); J.-Y. Ollitrault, *Phys. Lett. B* **273**, 31 (1991).  
 [30] I. Kvasnikova, C. Gale, and D. K. Srivastava, *Phys. Rev. C* **65**, 064903 (2002).  
 [31] D. K. Srivastava, *Phys. Rev. C* **71**, 034905 (2005).